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ANALYTIC CALCULATION OF THE ENERGY CHARACTERISTICS OF SLIP LINE FIELDS IN PLANE PLASTIC DEFORMATION PROBLEMS†

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An analytic implementation of the slip line method (the method of characteristics) is proposed for statically definable problems of the plane plastic deformation of an ideal rigid plastic medium. The solution of the Riemann problem (the initial characteristic problem) with boundary conditions defined by power series is represented in terms of generalized hypergeometric functions. Other boundary-value problems (the Cauchy problem and the mixed problem) reduce to the equivalent Riemann problems. A mixed problem with Prandtl friction in a curvilinear contact surface and a Cauchy problem with arbitrary smooth initial data are treated. An equation governing the form of the free surface is obtained. The analytic representations of the radii of curvature of the characteristics in the physical plane and in the plane of the velocity hodograph are used to calculate the dissipation power in the plastic domain. Problems of reduction in terms of long and short wedge-shaped matrices using the proposed energy approach are considered as an application. © 1997 Elsevier Science Ltd. All rights reserved.

Slip line fields in applied plasticity problems [1] are usually calculated using numerical or graphical methods [2–3]. In this case, there is no control over the accuracy of the calculations. Furthermore, the dissipation power is not calculated in this approach. A characteristic, the deformation stress, which is associated with this quantity and is important in applications is determined from the static conditions. Technical difficulties arise here in finding the arbitrary constant when integrating the stresses along an intersecting surface.

Exact solutions are known only for a small number of problems. The dissipation power has been found analytically [4] in the special case of fields which are formed by circular arcs. Below, this approach is extended to the case of arbitrary arcs which are defined by series in powers of the angular (characteristic) coordinates. The smoothness of the initial data enables us to obtain a solution of the Riemann problem in a convenient form from the well-known classical result [5]. The basic boundary-value problems is reduced to the equivalent Riemann problem for more general boundary conditions (a curvilinear boundary in the mixed problem and Cauchy problems with arbitrary smooth initial data) than in [6].

The results of the calculation of the stress in the case of a slip line field which is formed by circular arcs are identical to those obtained previously [4]. In the case of the problem of reduction in a long wedge-shaped matrix, the deformation stresses calculated using the proposed energy approach are compared with the values found approximately [6] from the static conditions.

1. BASIC RELATIONS

Suppose that a homogeneous isotropic body is in a state of plane plastic deformation: $\mathbf{u} = (u_1, u_2)$ is the velocity field, $(x, y) \in D$, D is the projection of the body onto the xOy plane; $\varepsilon(\mathbf{u}) = [\varepsilon_{ij}(\mathbf{u})]$ is the deformation rate tensor, $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$, $[\sigma_{ij}]$ is the stress tensor and σ is the hydrostatic pressure.

We assume that the following relations are satisfied in the domain D:

1. the equilibrium equation

$$\sigma_{ij,j} = 0, \quad i, j = 1, 2 \tag{1.1}$$

(the rule of summation over repeated indices is used)

2. the Mises flow condition (k is the plasticity constant)

$$(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2 = 4k^2$$

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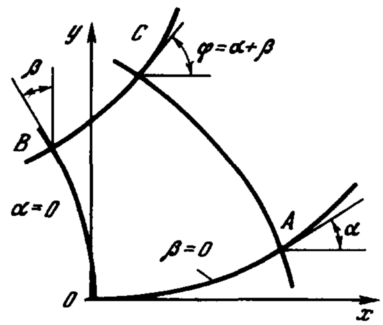

Fig. 1.

3. the incompressibility condition

$$\operatorname{div}\mathbf{u} = 0 \tag{1.2}$$

4. the condition for the direction of the area elements of maximum shear stress and maximum rate of shear strain to coincide

$$(\sigma_{11} - \sigma_{22})/\sigma_{12} = (\varepsilon_{11} - \varepsilon_{22})/\varepsilon_{12} \tag{1.3}$$

We know [7] that Eqs (1.1) reduce to a system of quasilinear equations of the hyperbolic type

$$\begin{aligned}&\partial\sigma/\partial x - 2k(\cos 2\varphi\ \partial\varphi/\partial x + \sin 2\varphi\ \partial\varphi/\partial y) = 0\\&\partial\sigma/\partial y - 2k(\sin 2\varphi\ \partial\varphi/\partial x + \cos 2\varphi\ \partial\varphi/\partial y) = 0\end{aligned} \tag{1.4}$$

where φ is the angle of inclination of an α-line to the abscissa. The characteristics of this system coincide with the slip lines. The Hencky conditions

$$\sigma - 2k\varphi = \text{const}, \quad \sigma + 2k\varphi = \text{const} \tag{1.5}$$

are satisfied along the α- and β-lines, respectively.

Equations (1.2) and (1.3) also reduce to a system of equations of hyperbolic type in the velocity components, and the characteristics of this system coincide with the slip lines.

In the domain where both families of characteristics are curvilinear, we define the characteristic coordinates (α, β) by means of the formulae

$$\alpha = \varphi/2 + (\sigma - \sigma_0)/(4k), \quad \beta = \varphi/2 - (\sigma - \sigma_0)/(4k)$$

where σ_0 is the value of σ at the origin of the system of coordinates. The geometric meaning of the coordinates (α, β) is clear from Fig. 1.

We introduce the following notation: $R(\alpha, \beta)$, $S(\alpha, \beta)$ are the radii of curvature of the α- and β-lines in the physical plane and $X(\alpha, \beta)$, $Y(\alpha, \beta)$ are the projections of the radius vector of the point $(x, y) \in D$ onto the direction of the slip line at this point. Suppose that (f_1, f_2) is any of the ordered pairs of (R, S), (X, Y) functions. The relations

$$f_1 + \partial f_2/\partial\alpha = 0, \quad f_2 - \partial f_1/\partial\beta = 0 \tag{1.6}$$

are then satisfied.

It follows from this that any of the above functions satisfies the telegraph equation

$$\partial^2 f/\partial\alpha\,\partial\beta + f = 0 \tag{1.7}$$

in the domain D.

2. SOME BOUNDARY-VALUE PROBLEMS

The Riemann problem. Suppose that the required function $f(x)$ is specified on the segments of the slip lines OA and OB (Fig. 1)

$$f(\alpha,0)=\sum_{n=0}^{\infty} c_n\alpha_n,\quad 0\le\alpha\le\alpha_1,\quad f(0,\beta)=\sum_{n=0}^{\infty} d_n\beta_n,\quad 0\le\beta\le\beta_1 \tag{2.1}$$

where c_n, d_n are given numbers and $\alpha_n = \alpha^n/n!$, $\beta_n = \beta^n/n!$. A solution of problem (1.7), (2.1) exists, and it is unique in the characteristic rectangle $OACB$ [8]. This solution is given by the formula [5]

$$f(\alpha,\beta)=c_0J_0(2\sqrt{\alpha\beta})+\sum_{n>0} c_n\frac{\partial^n I_0(2\sqrt{\alpha\beta}\,i)}{\partial\beta^n}+d_n\frac{\partial^n I_0(2\sqrt{\alpha\beta}\,i)}{\partial\alpha^n}$$

where $J_n(z)$, $I_n(z)$ are Bessel functions of real and imaginary argument, respectively.

Using functional relations for special functions [9], the last equality can be converted to the form

$$f(\alpha,\beta)=c_{00}F_1(1,-\alpha\beta)+\sum_{n=1}^{\infty}(c_n\alpha_n+d_n\beta_n)_0F_1(n+1,-\alpha\beta) \tag{2.2}$$

$$_0F_1(k+1,z)=k!\sum_{t=0}^{\infty} z_t\,/\,(t+k)!$$

Formula (2.2) remains true if the coordinate α and/or β are negative.

Now, suppose that the characteristics OA and OB are defined by their own radii of curvature

$$R(\alpha,0)=\sum_{n=0}^{\infty} a_n\alpha_n,\quad 0\le\alpha\le\alpha_1,\quad S(0,\beta)=\sum_{n=0}^{\infty} b_n\beta_n,\quad 0\le\beta\le\beta_1 \tag{2.3}$$

Using (1.6), we obtain the remaining boundary conditions

$$R(0,\beta)=a_0+\sum_{n=0}^{\infty} b_n\beta_{n+1},\quad S(\alpha,0)=b_0-\sum_{n=0}^{\infty} a_n\alpha_{n+1}$$

By virtue of (2.2), we obtain the solution of the Riemann problem for $R(\alpha, \beta)$, $S(\alpha, \beta)$ in the form

$$R(\alpha,\beta)=a_{00}F_1(1,-\alpha\beta)+\sum_{n=1}^{\infty}(a_n\alpha_n+b_{n-1}\beta_n)_0F_1(n+1,-\alpha\beta) \tag{2.4}$$

$$S(\alpha,\beta)=b_{00}F_1(1,-\alpha\beta)+\sum_{n=1}^{\infty}(b_n\beta_n-a_{n-1}\alpha_n)_0F_1(n+1,-\alpha\beta)$$

The Cauchy problem. Suppose the functions $\sigma = \sigma(\gamma)$, $\varphi = \varphi(\gamma)$ are given on the curve OC, which nowhere has a characteristic direction (Fig. 2). Henceforth, γ is the angle between the tangent to the curve OC and the Ox axis. A solution of system (1.4) with such initial data exists, and it is unique in the characteristic triangle OAC [8].

We now find the radii of curvature $R(\alpha, \beta)$, $S(\alpha, \beta)$ of the characteristics of the physical plane. From (1.5), the dependence

$$\sigma(\gamma) = \sigma_0 + 2k(\alpha(\gamma) - \beta(\gamma))$$

can be obtained in OC.

Since $\varphi(\gamma) = \alpha(\gamma) + \beta(\gamma)$, it is possible to determine $\alpha(\gamma)$, $\beta(\gamma)$ from this. With an accuracy up to an infinitesimal of higher order, we have

$$R(\alpha(\gamma),\beta(\gamma))\,d\alpha/\cos\eta = -S(\alpha(\gamma),\beta(\gamma))\,d\beta/\sin\eta = r(\gamma)\,d\gamma \tag{2.5}$$

where $r(\gamma)$ is the radius of curvature OC and $\eta = \gamma - \varphi(\gamma)$ is the angle between the tangent to OC and the α-line passing through the tangent point.

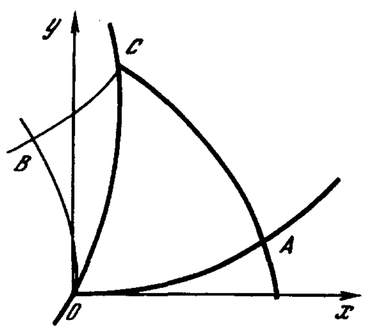

Fig. 2.

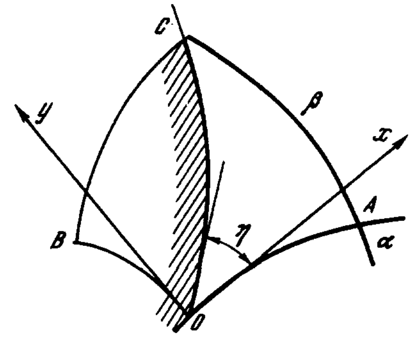

Fig. 3.

We assume that the functions $r(\gamma)$, $\alpha(\gamma)$, $\beta(\gamma)$ are sums of power series with uniformly bounded coefficients. From (2.5), it is then possible to obtain boundary conditions for the form of (2.3) for the equivalent Riemann problem.

We will now consider a special case. Suppose that the normal and shear stresses: σ_n = const, τ_n = const in OC are given. Then, in OC, we have

$$\alpha = \beta, \quad \eta = (\pi - \arccos(\tau_n/k))/2 = \text{const}$$

Denoting $r(\gamma) = r(2\alpha + \eta)$ by $r(\alpha)$, we obtain from (2.5) that

$$R(\alpha, \alpha)/\cos\eta = -S(\alpha, \alpha)/\sin\eta = \pm 2r(\alpha) \tag{2.6}$$

On substituting (2.4) here when $\alpha = \beta$, recurrence relations can be obtained for determining the coefficients a_k, b_k.

We now apply this scheme to a model problem concerning a body with a circular aperture of radius a loaded with a uniform normal pressure. Then, $\eta = \pi/4$. When $r(\alpha) = a$, we obtain the formula

$$a_0 = a\sqrt{2}, \quad b_0 = -a\sqrt{2}$$

$$\sum_{n+2t=k} (-1)^t \frac{a_n + b_{n-1}}{t!(n+t)!} = 0, \quad \sum_{n+2t=k} (-1)^t \frac{b_n - a_{n-1}}{t!(n+t)!} = 0, \quad k > 0$$

from (2.6) taking account of (2.4).

It follows from this that $a_n = a\sqrt{(2)}$, $b_n = (-1)^{n+1}a\sqrt{(2)}$ and hence $R(\alpha, 0) = a\sqrt{(2)}e^{\alpha}$, $S(0, \beta) = -a\sqrt{(2)}e^{-\beta}$, that is, the slip lines are logarithmic spirals. In applications, the case is important when $\sigma_n = \tau_n = 0$ in OC (the part of the boundary which is stress-free) but the equation of OC in the physical plane is unknown. If one of the characteristics is known, such as the α-line, for example, the radius of curvature of the curve OC

$$r(\alpha) = R(\alpha, \alpha)/\sqrt{2}$$

and the recurrence formulae for the coefficients b_n

$$a_n + b_{n-1} + b_n - a_{n-1} = 0, \quad n = 0, 1, \ldots$$

follow from (2.6) (the notation $a_{-1} = b_{-1} = 0$ has been introduced in order to simplify the writing).

The mixed problem. Suppose that a segment of the α-line OA and the angle $\varphi = \gamma - \eta(\gamma)$ are given on a smooth curve OC which nowhere has a characteristic direction: $x = x(\gamma), y = y(\gamma), \gamma \in [\gamma_0, \gamma_1]$ (Fig. 3). Such a problem arises, for example, if OC is a contact line in which the Prandtl friction law: $\tau_n = \mu k$, $0 \leqslant \mu \leqslant 1$ holds. We next assume that Prandtl friction acts in the contact line. Then, η = const. Suppose that $\beta = \beta(\alpha)$ is the equation of the curve OC in the $\alpha\beta$-plane. Then, by analogy with (2.6), we have in OC that

$$R(\alpha,\beta(\alpha))/\cos\eta = \pm S(\alpha,\beta(\alpha))\,\beta'/\sin\eta = \pm r(\alpha+\beta(\alpha)+\eta)(1+\beta') \tag{2.7}$$

We now consider some special cases.

1. OC is a segment of a straight line. Then, $\alpha + \beta = 0$ in OC. To be specific, suppose that $R(\alpha, \beta)$ and $S(\alpha, \beta)$ are of the same sign. As in the preceding case, we obtain the recurrence formulae for the coefficients b_n

$$(a_n + (-1)^n b_{n-1})\,\mathrm{tg}\,\eta - (-1)^n b_n + a_{n-1} = 0, \quad n = 0,1,\ldots$$

from the first equation of (2.7) taking (2.4) into account.

2. OC is an arc of a circle of radius a. Suppose that the equation of OC in the characteristic plane is $\beta = \beta(\alpha) = \Sigma_{k=0}^{\infty} c_k\alpha_k$.

Then, from (2.7), it is possible to obtain recurrence formulae for the coefficients b_n, c_k

$$a_0/\cos\eta = a(1+c_1), \quad -a_0\,\mathrm{tg}\,\eta = b_0c_1$$
$$(a_1 + b_0c_1)/\cos\eta = ac_2, \quad -(a_1 + b_0c_1)\,\mathrm{tg}\,\eta = (-a_0 + b_1c_1)\,c_1 + b_0c_2\ldots$$

A similar approach can be used in the case of an arbitrary curve under the assumption that $\beta(\alpha)$ is the sum of a power series.

Solutions of boundary-value problems for $X(\alpha, \beta)$, $Y(\alpha, \beta)$ can be obtained in an analogous manner.

3. PLASTIC POWER

Suppose that the plasticity domain D is the union of the domains D_i ($i = 1, 2, \ldots, l$), where D_i is one of the following domains: (1) a curvilinear characteristic rectangle; (2) a curvilinear characteristic triangle, the hypotenuse of which lies in the contact line; (3) a curvilinear characteristic triangle, the hypotenuse of which lies in the free surface; (4) the domain of a simple stressed state; 5) the domain of a uniformly stressed state [7].

We assume that the following conditions are satisfied.

1. Local Cartesian coordinates are defined in each domain D_i. We call the origin of the system of coordinates, the base point of the domain D_i. The Ox and Oy axes are directed along the α- and β-lines respectively, which pass through the base point. One of the local systems of coordinates is declared to be the global system of coordinates.

2. Local characteristic coordinates are introduced in each domain D_i. These are measured from the coordinate axes of the local Cartesian system of coordinates. The local characteristic coordinates and the radii of curvature of the characteristics preserve their sign in each domain D_i.

3. A tracking system of coordinates, the origin of which coincides with the base point, is associated with each point $M(\alpha, \beta)$ of a domain D_i and the coordinate axes are rotated through an angle $\alpha + \beta$ with respect to the axes of the local Cartesian system of coordinates.

4. Suppose that D_i is a curvilinear characteristic rectangle, C is a point which is diagonally opposite to the base point and D_i' is the image of the domain D_i in the plane of the hodograph. The image of the point C is the base point in D_i' and the local Cartesian system of coordinates is rotated relative to the tracking system at point C through an angle of $\pi/2$. The analogous relations for domains of other forms will become clear from the examples.

To be specific, if $D_i = \{(\alpha, \beta)\colon 0 \leqslant \alpha \leqslant \alpha_1, 0 \leqslant \beta \leqslant \beta_1\}$, it follows from 4° that the local characteristic coordinates (α', β') in the plane of the hodograph are related to the characteristic coordinates in the physical plane by the formulae

$$\alpha' = \alpha - \alpha_1, \quad \beta' = \beta - \beta_1.$$

The plastic power W in domain D is defined by the formula [7]

$$W/k = \sum_{i=1}^{l} \iint_{D_i} H\,dx\,dy + \sum_{j=1}^{m} \int_{l_j} |[u]_j|\,dl \tag{3.1}$$

where $H = \sqrt{(2\varepsilon_{ij}\varepsilon_{ij})}$ is the intensity of the shear rate, l_j ($j = 1, 2, \ldots, m$) are lines of discontinuity of the velocity and $[u]_j$ are the jumps in the velocity vector in the lines l_j.

Suppose that $\rho(\alpha', \beta')$, $\delta(\alpha', \beta')$ are the radii of curvature in the plane of the velocity hodograph. We know [3] that each of these functions satisfies the telegraph equation (1.7), in which the variables α, β can be replaced by α', β'. In the characteristic coordinates, we have [3]

$$\varepsilon_{11} = \varepsilon_{22} = 0$$
$$H = 2|\varepsilon_{12}| = |\rho(\alpha',\beta')/R(\alpha,\beta) - \delta(\alpha',\beta')/S(\alpha,\beta)|$$

where (α', β') is the image of the point (α, β) of the physical plane in the plane of the hodograph.

Analysis of the properties of the mapping of the mesh of characteristics in the plane of the hodograph when conditions 1–4 are satisfied shows that the expression within the modulus sign in the last formula preserves its sign in any domain D_i. This enables us to calculate the double integrals in (3.1) in the local characteristic coordinates.

Suppose, for example, that $D_i = \{(\alpha, \beta): 0 \leqslant \alpha \leqslant \alpha_1, 0 \leqslant \beta \leqslant \beta_1\}$. Then

$$W_i = \pm k \int_0^{\alpha_1} \int_0^{\beta_1} (\delta(\alpha - \alpha_1, \beta - \beta_1)\, R(\alpha,\beta) - \rho(\alpha - \alpha_1, \beta - \beta_1)\, S(\alpha,\beta))\, d\alpha\, d\beta$$

Here, $R(\alpha, \beta)$, $S(\alpha, \beta)$, $\rho(\alpha', \beta')$, $\delta(\alpha', \beta')$ are the solutions of the corresponding Riemann problems in the physical plane and in the hodograph plane which have the form (2.4). The last integral can easily be evaluated. Similar calculations can be performed if the domain D_i is the characteristic triangle in which the Cauchy problem or the mixed problem is formulated.

If D_i is the domain of a simple stressed state which, to be specific, is defined by the conditions $0 \leqslant \alpha \leqslant \alpha_1$, $\beta = 0$ then $S = \infty$. In this case it follows that the plastic power should be calculated using a polar system of coordinates matched to the slip lines (see Section 4).

4. EXAMPLES

1. Reduction of the band through a short wedge-shaped matrix. The deformation scheme and the corresponding hodograph are shown in Fig. 4. The radii R_0, S_0 are determined from elementary considerations and the quantity V_1 is determined by the incompressibility condition. The initial conditions for the Riemann problems have the form

$$R(\alpha, 0) = R_0, \quad S(0, \beta) = -S_0$$
$$\rho(\alpha', 0) = \delta(0, \beta') = \rho_0, \quad \rho_0 = |V_1 - V_2|/\sqrt{2}$$

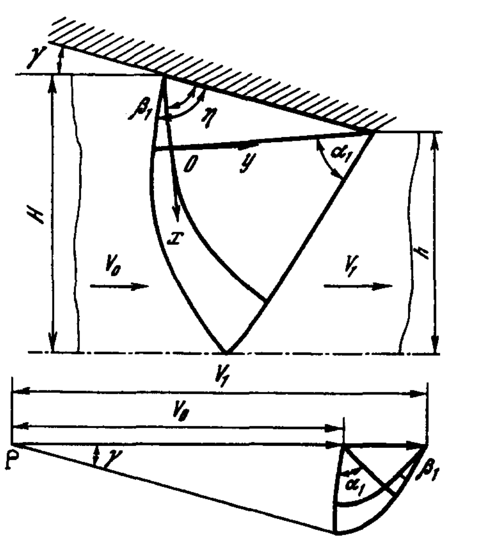

Fig. 4.

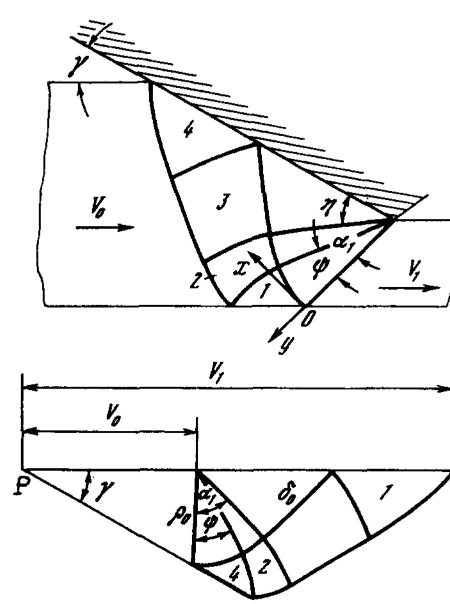

Fig. 5.

From (2.4), we have (in order to simplify the notation, the indices have been omitted in the notation ${}_0F_1$ for the hypergeometric function)

$$
\begin{aligned}
R(\alpha,\beta) &= R_0F(1,-\alpha\beta) - S_0\beta F(2,-\alpha\beta) \\
S(\alpha,\beta) &= -S_0F(1,-\alpha\beta) - R_0\alpha F(2,-\alpha\beta) \\
\rho(\alpha',\beta') &= \rho_0(F(1,-\alpha'\beta') + \beta' F(2,-\alpha'\beta')) \\
\delta(\alpha',\beta') &= \rho_0(F(1,-\alpha'\beta') - \alpha' F(2,-\alpha'\beta'))
\end{aligned}
\tag{4.1}
$$

On calculating the plastic power in the characteristic rectangle, we obtain

$$
\frac{W_1}{k} = \int_0^{\alpha_1}\int_{-\beta_1}^{0}(\delta R - \rho S)\,d\beta\,d\alpha = 2\rho_0(R_0(\alpha_1)_2\beta_1 + S_0\alpha_1(\beta_1)_2)F(3,\alpha_1\beta_1) +
$$
$$
+\rho_0(R_0 + S_0)(2\alpha_1\beta_1F(2,\alpha_1\beta_1) - F(1,\alpha_1\beta_1) + 1)
$$

We calculate the plastic power for the centred fan $D_2 = \{(\alpha,\beta): 0 \leqslant \alpha \leqslant \alpha_1, \beta$ in a polar system of coordinates. The pole of the polar system of coordinates is made to coincide with the vertex of the fan and the polar axis is directed along a radius. Since, $R(\alpha, r) = r$, $S = \infty$ then, using (1.6) and (4.1), we obtain

$$
\frac{W_2}{k} = \int_0^{R_0}\int_0^{\alpha_1}\rho(\alpha-\alpha_1,\beta_1)\,d\alpha\,dr = -R_0\int_0^{\alpha_1}\frac{\partial\delta}{\partial\alpha}d\alpha = R_0\rho_0(\alpha_1F(2,\alpha_1\beta_1) + F(1,\alpha_1\beta_1) - 1)
$$

An analogous expression is obtained for the second fan. On integrating the velocity jumps along the lines which bound the plastic domain and summing all of the resulting expressions, we find the power for the whole of the domain D

$$
W/(k\rho_0) = \alpha\beta(R_0\alpha + S_0\beta)F(3,\alpha\beta) + 2(R_0\alpha + (R_0+S_0)\alpha\beta + S_0\beta)F(2,\alpha\beta) + (R_0+S_0)F(1,\alpha\beta)
$$

where $\alpha = \alpha_1$, $\beta = \beta_1$.

Using the relations between hypergeometric functions and Bessel functions, the last formula can be reduced to the form

$$
W/(k\rho_0) = (R_0+S_0)(\xi_1 I_1(\xi_1) + I_0(\xi_1)) + 2(R_0\alpha_1 + S_0\beta_1)I_0(\xi_1), \quad \xi_1 = 2\sqrt{\alpha_1\beta_1}
$$

which is identical with the energy release formula from [4].

2. Reduction of the band in a long wedge-shaped matrix. The deformation scheme and the corresponding hodograph are shown in Fig. 5. By symmetry, the domain D_1 in the physical plane is formed by curved arcs of equal radii $R_0 = S_0$ and D_3 is a domain of the simple stressed state. The boundary β-lines in D_3 are equidistant curves. The boundary-value problems are successively solved using the results of Section 2: in the physical plane (in the domains D_1 and D_2 they are Riemann problems and in D_4 it is a mixed problem) and in the plane of the hodograph. In accordance with Section 3, the results of the calculation of the power can be represented in the form of repeated series.

For example, the plastic power in the domain D_1 is represented by series of the form

$$
\sum_{t,s=0}^{\infty} A_{ts}\psi_{t+s+1}\psi_s
$$

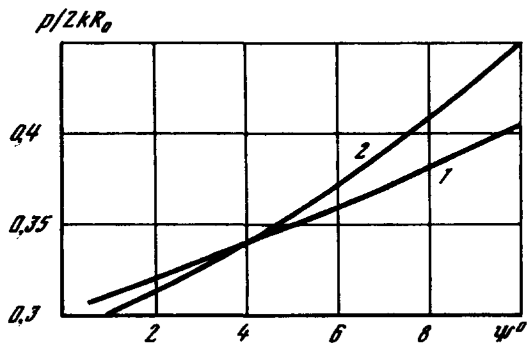

Fig. 6.

where the coefficients A_{ts} depend on the geometrical parameters of the plasticity domain and allow of the estimate

$$|A_{ts}| \le C2^t / (t-1)!$$

which is uniform with respect to s.

It can be shown that all the resulting repeated series are dominated by the exponential series. The results of calculations of the total deformation stress $p = W/N_0$, carried out with an accuracy of $\varepsilon = 10^{-5}$, are presented in Fig. 6 (curve 2) for the case of reduction with zero friction in the long matrix with a half-angle of conicity $\gamma = 10°$. For comparison, the results from [6], obtained with the same initial data from static conditions, are shown in the same figure (curve 1). The stress has been normalized with the factor $2kR_0$.

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